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I use the technique of Hernández, et al (hep-lat/0106011) to convert a recent calculation of the lattice-regulated quark condensate from an overlap action to a continuum-regulated number. I find $\Sigma_{\overline{MS}}(\mu = 2\text{ GeV}) = (282(6)\text{ MeV})^3 \times (a^{-1}/1766\text{ MeV})^3$ from a calculation with the Wilson gauge action at $\beta = 5.9$.

## I. INTRODUCTION

Studying lattice QCD with fermion actions which respect chiral symmetry without doubling (via the Ginsparg-Wilson relation [1]) is certainly a rewarding activity. In the past two years there have been two calculations of the (lattice-regulated) quark condensate $\Sigma$ in quenched QCD from overlap [2] actions [3,4]. Recently, Hernández et al [5] have adopted a matching of lattice and renormalization group invariant (RGI) masses to convert their lattice regulated result for the quark condensate into its RGI counterpart. The method involves determining the value of bare quark mass at which the pseudoscalar mass takes on a certain value and combining that bare mass with an appropriately-rescaled renormalization factor computed using Wilson fermions [6], called $U_m$ by the authors of Ref. [5]. The $Z-$ factor which converts the lattice quark mass to the RGI quark mass is then

$$\hat{Z}_M(g_0) = U_m \frac{1}{r_0 m_q}|_{(r_0 m_{PS})^2 = x_{ref}}. \tag{1}$$

Because Ginsparg-Wilson action are chiral, the renormalization factor for the condensate is $Z_S = 1/Z_M$, and the RGI condensate is

$$\hat{\Sigma} = \frac{\Sigma}{\hat{Z}_M}. \tag{2}$$

The RGI condensate can then be converted to the $\overline{MS}$ regulated condensate using a table of (multi-loop) conversion coefficients from Ref. [6]: $\Sigma_{\overline{MS}}(\mu) = \hat{\Sigma}/z(\mu)$ where $z = 0.72076$ for $\mu = 2$ GeV. My earlier calculation of $\Sigma$ [4] included a heuristic calculation of the lattice-to-$\overline{MS}$ $Z-$ factor, which the work of Ref. [5] renders obsolete. It is easy to use their results to update my old answer. The two fiducial points to which I can compare are $x_{ref} = 1.5376$ and 3.0, for which $U = 0.181(6)$ and $0.349(9)$ respectively.

The data set used in this study consists of 40 $12^3 \times 24$ lattices generated with the Wilson gauge action at $\beta = 5.9$. The nominal lattice spacing is $a \simeq 0.11$ fm, using the Sommer radius $r_0 = 0.5$ fm and the interpolating formula of Ref. [7] of $r_0/a = 4.483$. Spectroscopy has been computed using the overlap action of Ref. [4], at bare quark mass $am_q = 0.01, 0.02, 0.04, 0.06$. This is an action whose "kernel" is an approximate overlap action, with a connection to nearest and next-nearest neighbors and coupled to the gauge fields through APE-blocked [8] links. Coulomb gauge Gaussian shell-model sources and point sinks were used.

| $am_q$ | $r_0 m_q$ | $(r_0 m_{PS})^2$ | $(r_0 m_{PSS})^2$ | $(r_0 m_A)^2$ |
|---|---|---|---|---|
| 0.010 | 0.0448 | 0.853(118) | 0.332(82) | 1.088(203) |
| 0.020 | 0.0897 | 0.968(138) | 0.946(86) | 1.175(92) |
| 0.040 | 0.1793 | 1.928(144) | 2.164(82) | 2.022(78) |
| 0.060 | 0.2690 | 2.505(165) | 3.067(126) | 2.949(80) |

TABLE I. Pseudoscalar masses (scaled by $r_0/a = 4.483$) from the pseudoscalar correlator ($m_{PS}$), the difference of pseudoscalar and scalar correlators ($m_{PSS}$), and from the axial vector correlator ($m_A$).

The data are displayed in Fig. 1 and tabulated in Table I.

The overlap action has exact zero modes, which contribute a quark mass independent finite-volume lattice artifact in some channels, including the pseudoscalar (PS) and scalar channels [9]. The difference of pseudoscalar and scalar channels (which I will denote PSS below) receives no contribution from zero modes in both quark propagators. This is also the case for correlators of the axial current. The effect of the zero modes is to flatten the meson correlator and to give a meson mass in the PS channel which does not extrapolate to zero at zero bare quark mass. (In contrast, the PCAC quark mass extracted from this data set does not show any additive mass renormalization, as expected.) The PSS and axial channels do not suffer from this contamination. As a result of this difficulty, I have elected to use the PSS and axial data sets to interpolate quark masses from meson masses. At small quark masses the axial current decouples from the pion (since $\langle 0|\bar\psi\gamma_0\gamma_5\psi|\pi >= m_\pi f_\pi)$ and the signal in the channel degrades. Fortunately, the masses needed for Eq. 1 lie away from the smallest quark mass point.

The authors of Ref. [5] did not see any problems with zero modes, but their lightest quark mass is heavier than my two lightest ones.

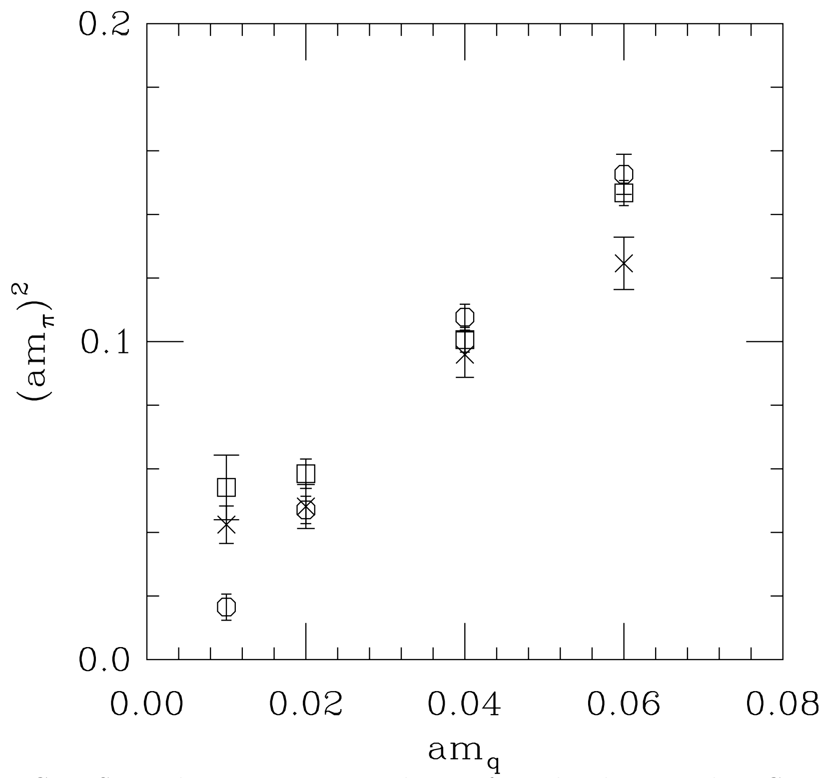


FIG. 1. Squared pion mass vs. quark mass from the planar overlap. Crosses are masses from the pseudoscalar correlator, octagons are from the difference of pseudoscalar and scalar correlators, squares from the axial vector correlator.

I extracted quark masses at the fiducial pseudoscalar masses via a combination of single elimination jacknife and $N-$point Lagrange interpolation, with $N = 2, 3, 4$: in each jacknife ensemble I fit the PSS and axial data sets, extracted pseudoscalar masses and interpolated them to find each $r_0 m_q$. The uncertainty on $r_0 m_q$ came from the jacknife. I found $r_0 m_q = 0.130(3)$ and $0.267(8)$ at $x_{ref} = 1.5376$ and 3.0, respectively. These results combine with the tabulated $U_m$'s to give $\hat{Z}_M = 1.392(56)$ and $1.307(52)$. I will assume that the difference between these two numbers is a statistical effect and combine them, continuing the analysis taking $\hat{Z}_M = 1.34(6)$. The uncertainty on this parameter is a bit smaller than authors of Ref. [5] found in their analysis; I believe that is due to the fact that I can bracket the smaller quark mass point, while in their case it is the smallest mass they measured.

The bare lattice regulated quark condensate from Ref. [4] is $\Sigma a^3 = 0.00394(16)$. Combining uncertainties in quadrature, I have an RGI condensate of

$$a^3\hat{\Sigma} = 0.00294(18) \tag{3}$$

or

$$r_0^3\hat{\Sigma} = 0.265(16). \tag{4}$$

Ref. [5] quotes 0.226(31) for the latter result, so we differ by just over one standard deviation. Of course, this is a comparison of two very different actions at one not-too-small lattice spacing.

Finally, taking the lattice spacing from $r_0$, $\hat{\Sigma} = 0.0162(10)$ GeV$^3$ or $(253(5)$ MeV$)^3 \times (a^{-1}/1766$ MeV$)^3$ and $\Sigma_{\overline{MS}}(\mu = 2$ GeV$) = (282(6)$ MeV$)^3 \times (a^{-1}/1766$ MeV$)^3$.

I have not included any uncertainty in $r_0$ in this result. The reader should be aware that using my extrapolation of the rho mass to set the lattice spacing would result in a lattice spacing of $a = 0.13$ fm rather than the result from the Sommer parameter of $a = 0.11$ fm.

The lattice-to-$\overline{MS}$ $Z$-factor for this action at $\beta = 5.9$ is $Z_M(\mu = 2$ GeV$) = 1.34(6) \times 0.72076 = 0.97(4)$. In Ref. [4] I had done a heuristic estimate of $Z_M \simeq 1.07$, based on experience with perturbation theory for clover fermions with fat-link actions [10]. Unpublished comparisons of perturbative calculations and nonperturbative measurements of additive mass renormalization and vector and axial current renormalization factors typically show values quite close to unity for large values of APE smearing, with deviations in the two determinations of $Z$'s of about 0.05 or so.

## ACKNOWLEDGEMENTS

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